# PROBLEMS IN PHYSICS.

THE following problems have been taken, for the most part, from DR. FLIEDNER'S work, entitled *Aufgaben aus der Physik.* They have been printed for the convenience of the classes in the Chemical Department of the Lawrence Scientific School of Harvard University.

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# PROBLEMS.

## EQUILIBRIUM BETWEEN LIQUIDS AND SOLIDS.

I. How many pounds avoirdupois do five cubic feet of water weigh? How many pounds Troy? How many kilogrammes?

II. How many ounces do five cubic inches of water weigh? How many grammes? How many grains?

III. What is the weight of six cubic inches of cast iron, its specific gravity being 7.21?

IV. What is the volume in cubic inches of 2 lbs. 12 oz. of brass, its specific gravity being 8? What in cubic centimetres?

V. What is the volume of 2 lbs. $6\frac{1}{2}$ oz. of gold, its specific gravity being 19.36?

VI. A leaden ball weighs in the air 8 lbs.; what is its weight in water, its specific gravity being 11.35?

VII. A man is just able to lift in water a stone one cubic foot in volume, and the specific gravity of which is 2.5; he is, however, unable to lift any portion of the same above the surface: how much can he lift out of water?

VIII. A vessel filled with water and placed on a scale pan is brought into equilibrium, and an insoluble solid not connected with the balance is so suspended as to be

entirely immersed in the liquid, yet without touching the bottom of the vessel: thereupon the scale pan supporting the vessel sinks, and, in order to restore the equilibrium, a certain weight must be placed on the other scale pan. If, now, this weight is $1\frac{1}{2}$ oz., what is the volume of the body immersed?

IX. A floating body, the volume of which is 36 cubic inches, is immersed in the water to one third of its volume. What is the weight of the body?

X. A homogeneous, prismatic body, 6 inches long, 4 inches wide, and 3 inches high, and weighing $1\frac{1}{2}$ lbs., is placed in water on its broader surface; to what depth will it sink?

XI. What, if the dimensions be 8 inches long, 5 wide, and 3 high, and the weight 2 lbs.?

XII. Kane observed in the Northern Seas floating masses of ice 2,000 feet long, 400 wide, and rising 200 feet above the surface of the water; what must have been the volume of ice under the water, assuming the weights of equal volumes of sea-water and ice to be to each other as 10 : 9?

XIII. With what force will a block of hemlock wood of three cubic feet in the water strive to ascend? (the specific gravity of hemlock being 0.6).

XIV. In order to raise a sunken ship, 20 empty and water-tight casks were employed, each 40 cubic feet in capacity and 300 lbs. in weight; these being attached to the vessel, she was made to float. What was the weight of the ship in water?

XV. In what proportion $x : y$ by weight must lead and cork be united in order that thus united they may float in water without any portion projecting above the surface, the specific gravity of lead being 11.35 and that of cork 0.24?

XVI. How much lead must be united to 1 lb. of cork

in order that the sum of their weights may equal the weight of an equal volume of water?

XVII. It is required to know the weight of a cork jacket capable of supporting in water a person weighing 130 lbs., the specific gravity of the human body being 1.1.

XVIII. It is required to construct a hollow sphere of sheet brass having the thickness $d = 0.5$ m. m. and a specific gravity $s = 8$, which shall sink just one half of its volume; what must be the outer and inner semi-diameters $R$ and $r$?

XIX. How deep will a wooden ball sink in water, its semi-diameter being $r = 5$ inches and its specific gravity $s = 0.4$?

XX. One pound of tin loses in water $1053\frac{1}{2}$ grains, one lb. of lead $674\frac{1}{3}$ grains, and 7 lbs. of an alloy of these two metals loses 6237 grains; what proportion of each metal is contained in the same?

XXI. The crown of the king of Syracuse weighed 20 lbs. and lost when weighed in water $1\frac{1}{4}$ lbs. Archimedes suspected the gold of this crown to have been mixed with silver. What must he have found to be the weight of each metal present, the specific gravity of gold being 19.26 and that of silver being 10.47?

XXII. The absolute weight of a body $= Q$, its specific gravity $= s_1$ and is greater than that of water. It is required to form such a mixture of this with a second body whose specific gravity $= s_2$ and is less than that of water, that the two bodies thus united may float in water under the same conditions as a third homogeneous body whose specific gravity $= s$. Required the weight $x$ of the second body.

XXIII. What weight of atmospheric air must a man weighing $Q$ lbs. attach to his body in order to float under the same conditions as hemlock wood, the specific gravity of the human body being 1.1, that of air 0.0013, and that of hemlock wood 0.6?

## DETERMINATION OF THE SPECIFIC GRAVITY OF SOLID AND LIQUID BODIES.

I. A piece of platinum has an absolute weight $G = 39.32$ oz., and a volume $V = 3$ cubic inches; what is the specific gravity $s$ of platinum?

II. The absolute weight of a piece of quartz being 3.24 oz., and its volume 2 cubic inches, what is its specific gravity?

III. $5\frac{1}{2}$ lbs of mercury have a volume of 10.58 cubic inches; what is the specific gravity of mercury?

IV. How may the specific gravity of an insoluble solid be ascertained by means of an ordinary balance?

V. By means of a hydrostatic balance the absolute weight of a body being found $= P = 161.875$ grammes, its weight in water $= P_1 = 99.375$ grammes; what is the specific gravity $s$ of the body?

VI. To determine the weight of a body that does not sink in water, it is united to another body of great specific gravity, as lead, e. g.; if now the absolute weight in the air of the body whose specific gravity is to be determined $= P$, the weight of the lead in water $= P_1$, and the weight of the two united together in water $= P_2$; what is the specific gravity sought?

VII. A piece of cork covered with varnish weighs in the air 30 grammes, a lead ball weighs in water 110 grammes, and when these two bodies are united they weigh in water only 15 grammes; what is the specific gravity of the cork?

VIII. An empty flask weighs two grammes, filled with sulphuric acid 38 grammes, filled with water 22 grammes; what is the specific gravity $s$ of sulphuric acid?

IX. What if filled with diluted sulphuric acid it weighed 31 grammes?

X. In a glass flask are 1.613 grammes of German sil-

ver; filled with water, the whole weighs 33.404 grammes; filled with water only, it weighs 31.981 grammes; what is the specific gravity of the german silver?

XI. By means of a hydrostatic balance, the loss in weight a body experiences in water being found to be 3 oz., while its loss in another fluid is $4\frac{1}{2}$ oz., what is the specific gravity $s$ of this fluid?

XII. What is the capacity $I$ of a vessel which empty weighs 1.26 lbs., filled with water 13.14 lbs.?

XIII. How great is the specific gravity $s$ of a mixture of several ingredients, when $G_1$, $G_2$, $G_3$, are the absolute weights, and $s_1$, $s_2$, $s_3$, the corresponding specific gravities?

### *Nicholson's Areometer.*

I. To determine the specific gravity of a diamond by means of a Nicholson's areometer, the diamond is placed on the plate, and by means of tare weights depressed to a certain line. The diamond is then removed and its weight replaced by 1.8 grammes, so as to sink the apparatus to the same line. Thereupon the added weights are removed, and the diamond placed in the basket at the bottom of the apparatus. There are now required 0.51 grammes to sink the apparatus to the observed line. Required the specific gravity of the diamond.

II. Upon what principle does the use of the weight-areometer (Nicholson's) rest, for determining the specific gravities of fluids? and upon what the use of the scale-areometer (Gay Lussac's) for the same purpose?

III. Upon a Nicholson's areometer 16 grammes must be laid to sink it to the mark in water, but only 1.1 grammes to sink it to the mark in a given sample of alcohol. If the weight of the instrument be 56 grammes, what is the specific gravity of alcohol?

## *Gay-Lussac's Areometer.*

I. A Gay-Lussac's volumeter (rational scale-areometer) consists in its simplest form of a tube of uniform calibre sealed at both ends, and one end made so much heavier that the tube swims erect in water. The tube is divided into equal parts from below up, so that in water the 100th division is exactly at the surface. In fluids heavier than water this division would be above, and in fluids lighter it would be below the surface of the water. Now placing this volumeter in four different fluids, it sinks to 56, 80, 125, and 140 of the scale; required the specific gravity of the corresponding fluids.

II. On account of the inconvenient length of a volumeter suited to fluids of extremely high and extremely low specific gravities, two sets have been made, one for fluids heavier than water, and the other for fluids lighter than water. Required the manner of making the scales.

## EQUILIBRIUM AND PRESSURE OF THE ATMOSPHERE.

I. How great is the pressure of the atmosphere upon a square inch, the barometer standing at 760 m. m.?

II. The barometer standing at the same height, what is the pressure, 1st, upon a rectangular table four feet long and three feet wide, 2d, upon a circular disk three feet in diameter, 3d, upon the surface of a sphere six inches in diameter?

III. What is the atmospheric pressure on the surface of the body of a full-grown man at the level of the sea, the height of the barometer being the same, this surface being 14 square feet? And what if the pressure be 731 m. m.?

IV. What height, $h$, must a column of water have, to balance the mean pressure of the atmosphere at the level of the sea?

V. What height a column of alcohol, and what a column of sulphuric acid?

VI. How great, expressed in atmospheres, is the pressure upon a pump-box above which a column of water stands 148 feet high?

VII. How many atmospheres' pressure is a fish subjected to at a depth of 300 feet below the surface of the sea?

VIII. The density of the air at the surface of the earth at 0° and 760 m. m. of the barometer is $\frac{1}{770}$ of the density of water; if now it were the same at all elevations, how high would the atmosphere extend? And why must the height of the atmosphere so calculated be found too small?

IX. What would be the height to which it would attain above the surface of the earth if it reached a point where the centrifugal force is balanced by gravitation?

X. What is the height of a column of air at 0° and 760 m. m. of the barometer, which shall balance a column of mercury 3 m. m. high?

*Mariotte's or Boyle's law.*

I. In order to test the correctness of Mariotte's law, we employ a glass tube of uniform calibre, bent like the letter U, but one of the arms very much longer than the other, the longer one being open and the shorter one closed. By pouring mercury into the longer arm the air in the shorter one is compressed; when now it is observed that the mercury in the shorter arm stands relatively at 4, 6, and 9 inches high, while at 760 m. m. the mercury in the longer branch stands at 18, 34, 93 inches, by what process, from these observations, do we arrive at the expression of Mariotte's law, the whole height above the mercury in the curved portion of the tube in the shorter arm being 12

inches, and the enclosed air, at the outset, having the same density as that of the air in the longer arm?

II. If under a barometric pressure of 760 m. m. the air in the shorter arm [12 inches long] is to be compressed to one half its volume, to what height must you pour the mercury in the long arm?

III. When at 760 m.m. the mercury stands at 24 inches in the longer arm, how high will it stand in the shorter arm, and to what part of its original volume will the air be compressed?

IV. When at 760 m.m. a tube 40 inches long, sealed at the bottom, is filled to a depth of 27 inches and inverted in a vessel of mercury, at what height will the mercury stand in the tube, and how many inches will the air occupy?

V. A gas mixture occupies under a barometric pressure $B = 27$ inches, a volume $V = 20$ cubic inches; how great is its volume $V_1$ by a barometric pressure $B_1 = 28$ inches?

VI. In a bell-glass, under a barometric pressure of 745 m. m., oxygen is collected over mercury, the surface of the mercury within the bell-glass is 7 m. m. above the surface of the mercury without; how great is the volume $V$ of this gas under a pressure of 760 m. m.?

*Barometer.*

VII. It may be derived from Mariotte's law, (neglecting the decrease in intensity of the force of gravity,) that in a uniformly warm column of air the barometric heights, and the forces of expansion measured by them, decrease in geometrical progression, while the heights in the air increase in arithmetical progression. A barometer standing at the surface of the sea at a temperature of 0°, 336 Paris lines (144 to a foot), falls one line when raised 73 feet,

what barometric heights, upon the supposition of uniform heat, uniform moisture of the air, and uniform intensity of gravitation, will be observed at elevations of $2 \times 73$, $3 \times 73$, $4 \times 73$, . . . $n \times 73$ Paris feet?

VIII. What under the same conditions would be the height $H$ of a place above the surface of the sea, where the barometer stands at 324 Paris lines?

IX. If the extreme confines of the air be taken at a point where the tension is balanced by a column of mercury of $\frac{1}{100}$ of a Paris line in height, what then, according to the conditions of the previous problems, would be the height $H$ of the upper limit of the air?

X. With the conditions of Problem VIII. required to determine the difference in height between two places, when the barometer stands at the lower at $B$ and at the upper at $b$?

More accurately than by the formula above given, the difference in height $H$ between the two places is found by the following formula from Gauss:—

$$H = C. (1 + 0.0026 \cos. 2\,\phi\ [1 + \tfrac{1}{399}\,(T + t)]\ (\log. B - \log. b)$$

in which

$C$ is a constant factor $= 56588$ for Paris foot, or $= 18382$ for metre,

$B$ the barometric height of the lower station,

$b$ " " " " upper station,

in any measurement only reduced to 0°.

$T$ the temperature of the air and the mercury at the lower station,

$t$ the temperature of the air and the mercury at the upper station,

$\phi$ the geographical latitude.

XI. When, now, by a determination of elevation in latitude 43° at the lower station, the barometric height $= 27''.\ 17$, the temperature of the air and the mercury $= 15°.\ 3\ R$, at the upper station, the barometric height

$= 19''. 845$, the temperature of the air and of the mercury $= 3°. 2\ R$, what will be the difference in elevation of the two stations in Paris feet ?

XII. What if the latitude were 48°, the barometric heights 754.123 m. m. and 701.275 m. m., and the temperatures respectively 24° C and 23°.5 C.

XIII. What if the latitude were 0°, the barometric heights 760 m. m. and 125.257 m. m., and the temperatures 30° C. and — 20° C.

## AIR BALLOON.

I. The volume of a balloon filled with gas $= P$, the weight of its cubical contents of atmospheric air at the surface of the earth $= a$, and that of its cubical contents of the gas with which the balloon is filled $= a_1$, the weight of the balloon itself [not including the gas] and the car attached and its contents $= Q$. How great is the ascending power, $P$, of the balloon ?

II. How great would be the volume $V$ of the balloon if it were merely to float in the air at the surface of the earth ?

III. How great must be the diameter $a'$ of a spherical balloon weighing 8 lbs. filled with pure hydrogen of spec. grav. $= 0.0693$, in order that the balloon shall rise ?

IV. The first air balloon (Charliere), with which the physicist Charles ascended from Paris in the year 1783, contained 10000 cubic feet, and $\frac{27}{28}$ of its contents at a barometric pressure of 28.3 inches were filled with impure hydrogen, which was only $\frac{4}{21}$ as heavy as atmospheric air, the weight of the empty balloon and the car, together with its contents, was 604.5 lbs. It is required to know,— 1. With what power the balloon ascends ? 2. At what height will it expand to its full ? And 3. How high can the balloon go ?

V. An air balloon has a volume $= V$, its weight and that of the car and contents $= Q$, and it is filled $\frac{1}{n}$ of its volume with a gas of spec. grav. $= s$. At what height $h$ will it be fully distended? and to what height $H$ can it ascend?

## EXPANSION OF BODIES BY HEAT.

### *Calculation of the Expansion by given Coefficients.*

I. How many degrees, according to Centigrade and Fahrenheit, are $n°$ Reaumur?

II. How many are $n°$ of Centigrade according to R. and Fah.?

III. How many are $n°$ Fah. according to R. and C.?

IV. The length of a body $= L$ at 0°, C and $= L_1$ at $t°$; what is its coefficient of expansion $a$?

V. The length of a body $= L$ at 0°, the coefficient of linear expansion $= a$; what length has the body at $t°$?

VI. The length of a bar of railway iron $= 8$ feet at 0°; how long is it at 40°?

VII. A bar of zinc has at 0° a length of 3 feet; how long is it at 25°?

VIII. The length of a body $= L$ at $t°$, the coefficient of expansion $= a$; what is its length at $t_1°$?

IX. A brass rod at 40° has a length of 5 feet; how long is it at 0°?

X. A cast-iron tube at 20° is ten feet long; how long is it at 100°?

XI. A glass rod at 25° has a length of 4 feet; how long is it at 400°?

XII. How long must a brass rod be at 15° that at 0° it may be exactly 2 feet long?

XIII. A rectangular plate has at 0° a superficies $= F$; how great will it be when the temperature is raised to $t°$, the coefficient of linear expansion being $= a$?

XIV. A rectangular plate of cast-iron has at 0° a superficies of 20 square feet; how great is it when heated to 80°?

XV. A brass circle has at 0° a radius of 5 inches; by how much is its superficies greater at 36° than at 0°?

XVI. A rectangular body has at 0° solid contents $= K$; how great is it at $t°$ with a coefficient of linear expansion $= a$?

XVII. A block of sandstone at —10° has a volume of 12 cubic feet; what will be its volume when heated to 40°?

XVIII. A brass sphere has at 0° a diameter = 0.5 of an inch; how great is it as 100°?

XIX. A quantity of mercury occupies a space of 5 cubic inches at 0°; what space at 80°?

XX. A glass globe contains at 10° a cubic foot of water, how great will be its contents when filled with water at 90°?

XXI. What space will 25 cubic inches of mercury at 15° occupy, when heated to 100°?

XXII. The density of mercury at 0° = 13.6; what is it at $t°$?

XXIII. The height of the mercury in a barometer is observed at $t° = H$; how great is it at 0°; and how great the correction to reduce this barometric height $H$ to the normal temperature 0°?

XXIV. Reduce the observed barometric height of 764.4 m.m. at 20° to 0°.

XXV. In accurate barometric observations, it is necessary, in addition to the correction for the expansion of the mercury by heat, to make a correction for the metallic scale reaching from the lower to the upper surface of the mercury; now if $t$ be the temperature indicated upon the scale at which its graduation was made, and $L$ its length or the barometric height reduced to 0°, and $a_1$ the coeffi-

cient of linear expansion of the metal of the scale; how great will be its length $x$ at $t°_1$, and how great the correction to be applied to the barometric height?

XXVI. How many less vibrations will an iron pendulum of the length equal to 3.167 feet daily make, which at a given temperature beats seconds, when the temperature is raised 20°?

XXVII. In a compensation pendulum in which the length of the pendulum at $t° = L$, the coefficient of expansion of the metal whose expansion increases the length $= a$, and that of the metal whose expansion lessens the length $a = a_1$; how great must the length $x$ of the latter be in order that the influence of heat may be compensated?

XXVIII. What must be the length of the bars expanding upward, when of zinc, the bars expanding downward being of iron, and the pendulum 2 feet long?

XXIX. What if brass be used instead of zinc?

XXX. A Graham compensation pendulum consists of a glass rod to which a glass vessel containing quicksilver is attached, the length of the pendulum being $= L$; how high, expressed in parts of $L$, must the quicksilver stand in the glass vessel that the compensation may be perfect?

XXXI. A mass of water which at 0° has a volume $= V$ has, according to Hallström, at $t°$ a volume

$$V_1 = V(1 - 0.000057577\, t + 0.0000075601\, t^2 - 0.000000035091\, t^3),$$

when $t$ is between 0° and 30°; but when $t$ is between 30° and 100°,

$$V_1 = V(1 - 0.000094178\, t + 0.0000053366\, t^2 - 0.000000010409\, t^3).$$

According to this, how great is $V_1$ at 11.5°, how great at 8.9°, and how great at 50°?

XXXII. A glass with a narrow neck filled with water

at 0° weighs, independent of the glass, 124 grammes. After heating the vessel to 100°, 6 grammes of water ran out. From these data what is the expansion of a unit of volume of water, the coefficient of cubical expansion of the glass being taken at 0.000026 ?

### *Expansion of Gases.*

I. The volume of a gas at 0° $= V$; how great is it at $t°$ ?

II. The volume of a gas at $t° = V$; how great is its volume at $t°_1$, the pressure remaining the same ?

III. How does the volume change when the pressure passes from $P$ to $P_1$ ?

IV. What space will 5 cubic feet of air at 0° take when heated to 100°, the pressure remaining the same ?

V. What if the temperature at the outset was 20° ?

VI. A mass of air of 800 cubic feet, of 15 lbs. tension (upon the square inch) at 10°, heated in a furnace to 200° acquires a tension of 19 lbs.; what space does it occupy ?

When a gas of a volume $V$ is by heat expanded to $V_1$, and the pressure acting upon the gas changes, the following relations between the volumes $V$, $V_1$, their respective temperatures $t$, $t_1$, and the pressures $P$, $P_1$, are observed, $\frac{V_1}{V} = \frac{P}{P_1} \times \frac{1 + a\,t_1}{1 + a\,t}$; or as when $E$, $E_1$ indicate the tensions corresponding to the pressures $P$, $P_1$, $\frac{E_1}{E} = \frac{P_1}{P}$, $\frac{E_1}{E} = \frac{V}{V_1} \times \frac{1 + a\,t_1}{1 + a\,t}$, in which expression in degrees of the Centigrade $a = 0.003665 = \frac{1}{273}$, and in degrees of Reaumur $a = 0.004581 = \frac{1}{218}$.

With the aid of these formulæ the value of any one of the quantities therein occurring may be found when the others are given, and the following problems may be solved.

VII. What pressure $P_1$ is necessary in order that a measured volume of gas $V$ under a pressure $P$ and at a temperature $t$ shall at a temperature $t_1$ go over into a volume $V_1$?

VIII. How great is the tension $E_1$ of a gas which under the volume $V$ at the temperature $t$ possesses the tension $E$ when by heating to the temperature $t_1$ it acquires the volume $V_1$?

IX. How great is the tension $E_1$ of this gas when raised to the temperature $t_1$, if its volume cannot change because it is enclosed in a vessel of permanent size?

X. What temperature $t_1$ will a gas having a volume $V_1$ and a pressure $P_1$ acquire, when it has with a volume $V$ and under a pressure $P$ the temperature $t$?

XI. A glass globe, the neck of which is closed by a cock, is filled with a gas having a temperature $t$ and a pressure $H$; required the method of obtaining the weight of this gas at 0° under a pressure of 760 m. m.

XII. Required the capacity of a glass balloon which at 20.5° and 755 m. m. filled with dry air weighs 567.949 grms., filled with pure water of the same temperature weighs 6133.162 grms., and exhausted of its air to a pressure of 5 m. m. weighs 561.291 grms.

## CONSTRUCTION OF THERMOMETERS AND DETERMINATION OF THE COEFFICIENTS OF EXPANSION.

I. Required the interior capacity $V$ of a narrow glass tube of uniform calibre at a temperature $t$.

II. How may a narrow glass tube of unequal calibre be divided into parts of equal capacity?

III. Having divided into parts of equal capacity a glass tube on one end of which a bulb is blown, required how many times the capacity of the bulb is greater than one division of the tube.

IV. Required a mercurial thermometer by which temperatures from 200° above to 40° below zero, may be determined; how many times must the capacity of the bulb be greater than that of the tube?

V. A glass tube is divided into 260 parts of equal interior capacity, and when weighed first empty, and then with a column of mercury occupying 40 divisions, the difference is found to be 2 grms. Required the semi-diameter of the spherical bulb, in order that the 260 divisions of the tube may form 180° of the Centigrade.

VI. The boiling point of a thermometer is only then right when the height of the barometer observed at the same time reduced to 0° is 760 m. m.; for every other barometric height a correction is necessary. Assumed that the barometric height is 753 m. m., and the observed distance of the boiling point from the melting point of ice $= e$, how great will be the corrected distance $e_1$?

VII. Through the medium of a weight thermometer (called also air thermometer), which consists of a glass tube 4 inches long and 1 inch wide, one end of which is drawn out to a fine narrow tube, the temperature of the vapor of boiling sulphur is to be determined. The mercury required to fill the apparatus is found to be 610 grms. After filling the tube with dry air and then exposing it to the vapor of sulphur till no more air escapes through the fine opening, the fine tube is closed with the blowpipe. After cooling the tube to 13° and breaking off the point under mercury, 317 grms. of mercury pass into the tube, when the inner level of the mercury and the outer level are the same; required the temperature $x$ of the vapor of sulphur.

VIII. Borda's Thermometer (Pyrometer) consists of two rods, one of platinum and the other of copper, which rest one upon the other, and at one end are fastened together. The free end of the copper rod, which is some-

what shorter than the platinum, serves as an index, and the graduation is accomplished by plunging the instrument successively into melting ice and boiling water, and the position of the index upon the platinum rod indicated by two marks; how great will be the intervening space $x$ between the two marks when the copper rod is five feet long?

IX. It is required to determine the apparent expansion of mercury,—

*a*, by means of a glass tube divided into parts of equal interior capacity with a spherical bulb containing $n$ times the space of one division of the tube;

*b*, by means of a weight thermometer.

X. The coefficient of the apparent expansion of mercury $= \frac{1}{6480}$ that of the absolute expansion $= \frac{1}{5550}$ for the Centigrade thermometer; required the coefficient of expansion of glass.

XI. In order to determine the coefficient of expansion of platinum or of iron the following process is employed; a rod of metal weighing $P$ is placed in a short glass tube, of which one end is hermetically sealed and the other drawn out to a fine point. After filling the apparatus with mercury, the weight of which $= P_1$, it is placed in a horizontal position in a vessel filled with oil, so arranged that the fine tube protrudes through the wall of the vessel; the oil vessel is then heated till the temperature of the apparatus rises from $t$ to $t_1$, by which a quantity of quicksilver is forced out, and on being collected found to weigh $P_2$. If now the density of the metal $= d$, the coefficient of expansion of quicksilver $= D$, its density $= d_1$, and the coefficient of the expansion of the glass $= K$; required the equation for determining the coefficient of expansion $C$ of the metal.

XII. A rod of metal, the expansion of which by heat is to be measured, rests with one end against a fixed object, with the other against the short arm of a lever, the shorter

arm having a length $= l_1$, while the longer arm (the pointer) has a length $= l_2$; if now the metallic rod at a temperature $t$ has a length $= L$, and by heating it to $t_1$ the pointer is turned around through the angle $w$, what is the coefficient of expansion of the metal?

XIII. To find the expansion of water a glass tube drawn out to a fine point has been filled with water at 0°, and weighs (independent of the glass) 186 grammes. When now by heating to 100°, 9 grammes of water are driven out, the cubical coefficient of expansion of glass being = 0.000026, how great is the expansion of the water from 0° to 100°?

XIV. To determine the coefficient of expansion of a gas, a graduated glass tube with its bulb is filled with dry gas, and heated to 100°; then observing the height of the barometer $b$, it is hermetically sealed. After the apparatus is cooled to 0°, the previously sealed point is broken off under mercury, the height $B$ to which the column of mercury rises in the tube, and the height of the barometer $b_1$ being determined, the opening of the tube is stopped with wax, and the weight of the mercury in the tube $= p$ determined, and also the weight of the mercury which at 0° fills the whole tube and bulb $= p_1$; it is required from these observations to determine the coefficients of the apparent and absolute expansion of the gas.

## DETERMINATION OF THE SPECIFIC GRAVITY OF BODIES, TAKING INTO ACCOUNT THEIR EXPANSION BY HEAT.

I. With the aid of a hydrostatic balance the weight of a solid body is found $= P$ and its loss of weight in water $= p$ at a temperature $t$: what is its specific gravity at a temperature 0°, and compared with water at its greatest density 4°?

II. The weight of a given fluid $= P_1$ at a temperature

$t_1$, and the weight of an equal volume of water $= P$ at a temperature $t$, both fluids having been weighed in the same flask. What is the specific gravity of the first fluid at 0° referred to the greatest density of water?

III. A balloon of exactly 10 litres' capacity is found to weigh at 18° and 754 m. m. barometer 12.01 grammes more when filled with air than when so far exhausted as to have a tension of 5 m. m. barometer. What is the density of the air at 0° and 760 m. m. of the barometer?

IV. A gas has, under a temperature $t$ and under a pressure $b$, a volume $= V$, and a weight $= P$; what is the density of this gas, that of air at 0° and 760 barometer taken as unity?

V. What is the density of air at $t°$, and a barometric pressure of $b$, when referred to water?

VI. What is the weight of a cubic foot of atmospheric air at a temperature $t$, and a barometric pressure of $b$?

## SPECIFIC HEAT AND CHANGE OF MOLECULAR ARRANGEMENT OF BODIES.

I. If the quantity of heat necessary to raise the temperature of 1 pound of water 1° be taken as unity, what is the quantity necessary to raise $m$ pounds of water from 0° to $t°$?

II. If two masses $m_1$, $m_2$ of the same fluid of the temperatures respectively of $t_1$, $t_2$ be mixed together, what equation will give the temperature $t$ of the mixture?

III. If 7 lbs. water at 25° be mixed with 3 lbs. water at 65°, what will be the temperature of the mixture?

IV. What will be the temperature of a mixture of 6 lbs. mercury at 20°, and 4 lbs. of mercury at 50°?

V. 10 lbs. iron turnings at 100° are mixed with 18 lbs. water at 15°, and thereupon a temperature of 20° observed. From this result what do we find to be the specific heat of iron?

VI. One lb. mercury at 100° and one lb. water at 7° give a mixture at 10°; required the specific heat of mercury.

VII. Two masses $m_1$ and $m_2$ of different substances have temperatures $t_1$ and $t_2$ and specific heats of $w_1$ and $w_2$. The temperature of the mixture $= t$; what equation will give the value $t$?

VIII. What will be the temperature of a mixture of 8 lbs. water at 100°, with 10 lbs. mercury at 22.5°?

IX. Of two substances one has a temperature $t_1$, and a specific heat $= w_1$; the other has a temperature $t_2$, and a specific heat $= w_2$. A mixture of $p$ lbs. is to be made, having a temperature $t$; how many pounds of each substance will be required?

X. By the method of mixtures to ascertain the specific heat of a body, it is necessary to know the specific heat of the vessel. To find this a quantity of water $= M$ is poured into the vessel, which is, for example, of copper, and has a mass $= m_1$, and the common temperature is observed to be $t_1$. Then another mass of copper $= m_2$ of a temperature $t_2$ is taken, and the temperature of the mixture $t$ noted. From these data required the specific heat $w$ of the copper.

XI. In order to determine the specific heat of copper by the method of Laplace and Lavosier, a sphere of copper weighing three pounds and heated to 100° C. was placed in the calorimeter, upon which 0.38 lb. water flowed out. From these data what is the specific heat $w$ of copper?

[The latent heat of water according to more recent researches is 79.25° C.; according to old determinations, 75° C.]

XII. 5 lbs. iron of 80° C. melted in the calorimeter 0.57 lb. ice; what is the specific heat of iron?

XIII. What temperature will a mixture of one pound of ice at 0° and one pound of water at 80° take on?

XIV. By mixing 3 lbs. ice at 0° with 7 lbs. water at 100° C. there is produced water of 46.2°; how great is the latent heat in each pound of ice?

*Solution.* — If we take the quantity of heat which is necessary to raise the temperature of 1 lb. of water 1°, as the heat unit, we have for 7 lbs. water at 100° C. $7 \times 100$ heat units and for 10 lbs. water produced after the mixture of 46.2° C. $10 \times 46.2$ heat units. To melt the 3 lbs. ice there were required $700 - 462 = 238$ heat units, or for each pound $\frac{238}{3} = 79.3$ heat units: that is, the same quantity that is required to raise the temperature of 1 lb. of water to 79.3 C.; — or when $x$ designates the number of degrees sought, then from $3x + 10 \times 46.2 = 7 \times 100$, the value of $x = 79.3$ C.

XV. What temperature will be produced by pouring 4 lbs. boiling water upon 3 lbs. ice at 0°?

XVI. How many lbs. snow at 0° must be mixed with 6 lbs. water at 95° C. in order to have the resulting water at 10°?

XVII. If you expose ice at 0° to a constant source of heat of such intensity that 8 minutes are required to change it to water at 0°, experience teaches that from that point on, in ten minutes the water will be heated to 100°, and at this temperature it remains through the following 54 minutes, when all the water will be converted into steam; required from these observations the latent heat of water and the latent heat of steam.

XVIII. By conducting the steam of 1 lb. of water into 5.4 lbs. of water at 0°, what will be the temperature of the 6.4 lbs. water produced?

XIX. If you conduct the vapor of 4 lbs. of boiling water into 60 lbs. of water at 0°, to what temperature will it be brought?

XX. But if 60 lbs. water instead of having the temperature of 0°, have a temperature of 16°, what will the temperature of the mass be in that case?

XXI. How many lbs. steam at 121° C. are required to heat 300 lbs. water from 11° to 28° C.?

XXII. To what temperature must tin be heated above its melting point in order that the latent heat required in melting be employed only to elevate the temperature?

XXIII. How many lbs. sulphuric ether will be raised from 0° to the boiling point (35.6° C.) by the heat necessary to evaporate 1 lb. of ether?

## DENSITY, VOLUME, AND EXPANSIVE FORCE OF VAPORS AND PARTICULARLY OF THE VAPOR OF WATER.

I. It has been found by experiment that steam at 100° C. and 760 m. m. of the barometer occupies a space 1700 times as great as water at 0° C.; also that atmospheric air at 0° C. and the same height of barometer fills a space of 770 times greater volume than an equal weight of water at 0° C. What is the density of steam compared with that of air at 100° C. and 760 m. m. pressure?

II. 1 volume of oxygen and 2 volumes of hydrogen combine to form 2 volumes of aqueous vapor of the same temperature and tension; what is the density of aqueous vapor compared with that of atmospheric air?

III. Observation has shown the tension of aqueous vapor at 0° to be 5.059 m. m.; what is its density?

IV. What is the density of aqueous vapor at 40° C. in saturated condition?

V. How much does a cubic foot of aqueous vapor weigh at 100° C.?

VI. What is the weight of a cubic foot of aqueous vapor at 25° C.?

VII. What will be the tension of vapor of water at 100° C. heated out of contact with water to a temperature of 121° C. without the possibility of expansion?

VIII. What space will 1700 cubic feet of aqueous vapor

of 100° C. occupy, if heated out of contact with water to 121° C., the vapor being free to expand, that is, the pressure remaining constant?

IX. Why does the expansive force of steam in contact with water increase more rapidly than the temperature of the space which it fills?

X. What is the density of saturated steam at 121° C.?

XI. What space will be required for the steam saturated at 121°, of one cubic foot of water?

XII. How great is the density of saturated aqueous vapor under a tension of five atmospheres? And how much does a cubic foot of such aqueous vapor weigh?

XIII. How great is the expansive force of aqueous vapor saturated at 110°?

XIV. What is the pressure upon each square inch of the inner surface of a closed vessel in which water is heated to 166.5°? *

XV. How is it to be explained that saturated aqueous vapor of high tension, that is, of temperature above 100° when streaming out from a cock, has a temperature near the opening much below 100°?

XVI. It has been found in a thermometric measurement of heights that the water at the lower station boiled at 99.5°, and at the upper at 97°; how many feet is the last station higher than the first?

XVII. To determine the density of a vapor (by the process of Dumas), a glass balloon, the neck of which is drawn out to a point, is taken, having a volume of 10000 c. c.; it is first filled with air at 0°, and its weight found to be = 1015 grammes, then the fluid, the vapor of which is to be determined, is placed in the balloon, which is then

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* The expansive force is calculated from Holzman's interpolation formula, $\log. p. = 0.656 + \frac{7.4804\,t}{236.22 + t}$, in which $p$ represents millimetres and $t$ degrees of the Centigrade.

immersed in a vessel containing heated oil until all the fluid is changed to vapor. The opening at the finely drawn point is then closed with the aid of a blow-pipe. The weight of the balloon is now 1010 grammes. How great is the density of the vapor referred to that of atmospheric air at 0° ?

XVIII. How great is the weight of the aqueous vapor in a cubic foot of atmospheric air perfectly saturated at 25° ?

XIX. How great when the air is not saturated with aqueous vapor, but the dew point lies at 12° ?

XX. At a temperature of 24° the dew point was found by a Daniell's Hygrometer to be at 13° ; how many per cent is this vapor of the total which the air could contain at perfect saturation ?

XXI. It is required to calculate the quantity of rain that would fall from 1000 feet of height upon a quarter of a square mile, the air being saturated with aqueous vapor at 20°, and its temperature sinking to 11°.

XXII. What would be the quantity of rain, upon the supposition that the air was not saturated with aqueous vapor at 20°, but that the dew point of the air was at 15° ? Required the height of clouds, the air at 20°, and the dew point at 15°.

## COMBUSTION AND WARMING.

I. How many calorics (units of heat) will be produced by the combustion of five pounds of pure coal ?

II. When one cord of air-dried wood weighs 3000 pounds and costs $ 12, while 1000 pieces of turf weighing 700 pounds cost but $ 2.33, what are the relations of the values for heating purposes of the two kinds of fuel ?

III. According to many experiments, it is estimated that one pound of good stone-coal will on an average, with a well-arranged grate and boiler, evaporate 6 pounds of water. It is required to estimate the heat that is lost.

IV. Required to estimate the amount of stone-coal necessary to evaporate in a steam-boiler 10 cubic feet of water hourly.

V. How many cubic feet of atmospheric air are required to burn 1 pound of stone-coal, when the stone-coal contains $\frac{6}{7}$ of its weight of pure carbon?

By the word *heat effect* of a combustible is understood the quantity of heat expressed in calorics [or heat units] which will be evolved by the perfect combustion of a definite weight of the combustible.

VI. Required the heat effect $a$ by the combustion of a carbo-hydrogen compound, compared with that of carbon $= 1$, when in the unit of weight of the compound $w$ parts of hydrogen, and $k$ parts of carbon are contained.

VII. The heat effect $a$ of a compound is to be estimated which contains in a unit of weight $w$ parts of hydrogen, $k$ parts of carbon, and $s$ parts of oxygen, but in which the oxygen or a part of it must be considered as combined with hydrogen forming water.

VIII. Alcohol consists of 52.66 per cent of carbon, 12.90 per cent of hydrogen, and 34.44 per cent of oxygen, and its chemical formula is $C_4\,H_6\,O_2$; required the heat effect $a$ of alcohol.

IX. When one compares the weights of oxygen which equal weights of hydrogen and carbon require to convert them into carbonic acid and water, it is found that they stand in the same relation as the heat effects of carbon and of hydrogen to wit, as 1 : 3. From this it follows that one part by weight of oxygen uniformly gives out in combustion the same amount of heat; how great is it?

X. In order to ascertain by Berthier's process the heat effect of a sample of stone-coal, one gramme is taken and finely pulverized, mixed with a great excess of oxide of lead and ignited in a crucible. After perfect combustion of the carbon, there are found $n$ grammes of metallic lead; required from this the heat effect of stone-coal.

XI. Required the combustion temperature of carbon by its combustion in pure oxygen.

XII. The same of hydrogen.

XIII. The same of carbon by its combustion in atmospheric air.

XIV. The same of hydrogen.

XV. How is it to be explained that the combustion temperature of carbon is more than three times as great as that of hydrogen, though in relation to the heat effect of these substances the reverse is true?